# Advanced structural dynamics

for given problem :-

$$[m] = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & 2m \end{bmatrix} \qquad K = \begin{bmatrix} 3K & -2K & 0 \\ -2K & 4K & -2K \\ 0 & -2K & 2K \end{bmatrix}$$

$$F = \begin{Bmatrix} 0 \\ 0 \\ f(t) \end{Bmatrix}$$

The equations of Motion are

$$m\ddot{x}_1 + Kx_1 + 2K(x_1 - x_2) = 0$$

$$m\ddot{x}_2 + 2K(x_2 - x_1) + 2K(x_2 - x_3) = 0$$

$$2m\ddot{x}_3 + 2K(x_3 - x_2) = F(t)$$

To write the equation in State space form :-

$$y_1 = x_1 \qquad \dot{y}_1 = y_2$$

$$y_2 = \dot{x}_1 \qquad \dot{y}_2 = -\frac{1}{m}[Ky_1 + 2K(y_1 - y_3)]$$

$$y_3 = x_2 \qquad \dot{y}_3 = y_4$$

$$y_4 = \dot{x}_2 \qquad \dot{y}_4 = -\frac{1}{m}[2K(y_3 - y_1) + 2K(y_3 - y_5)]$$

$$y_5 = x_3 \qquad \dot{y}_5 = y_6$$

$$y_6 = \dot{x}_3 \qquad \dot{y}_6 = \frac{1}{2m}(F(t) - 2K(y_5 - y_3))$$

Now writing in State Space form

$$\begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \\ \dot{y}_3 \\ \dot{y}_4 \\ \dot{y}_5 \\ \dot{y}_6 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -\frac{3K}{m} & 0 & \frac{2K}{m} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ \frac{2K}{m} & 0 & -\frac{4K}{m} & 0 & \frac{2K}{m} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & K/m & 0 & -\frac{K}{m} & 0 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \end{bmatrix}$$

$$+ \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{2m} \end{bmatrix} F(t)$$

Now other

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \end{bmatrix}$$